

**Education &  
Training**



# Radars

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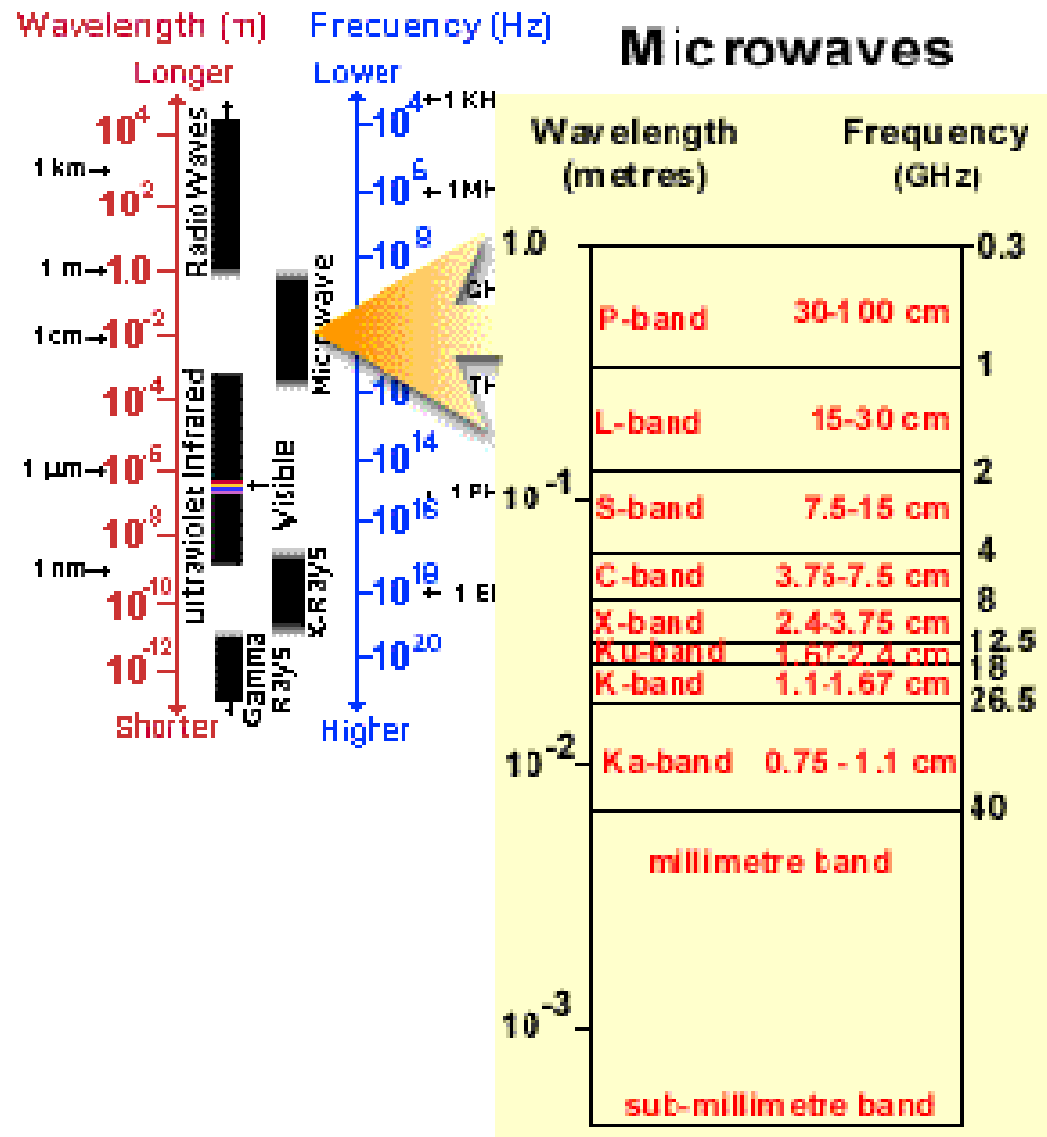
*TETRAD training course, 10-18 September 2010, Hyères, France*

# **RADAR (RAdio Detection And Ranging)**

- ❖ **Active remote sensing instrument (like LIDARs)**
  - ❖ **First meteorological applications in 1935**
  - ❖ **First pulsed RADAR during WWII**
  - ❖ **First Doppler application in the '70**
  - ❖ **First polarimetric applications in 1976**
  - ❖ **Operative networks established in the '80**
- 
- ❖ **To detect presence, position, speed of objects (targets) by radio (cm to m) or micro (mm to cm) waves**
  - ❖ **For meteorological RADARs the target are cloud particles and hydrometeors**

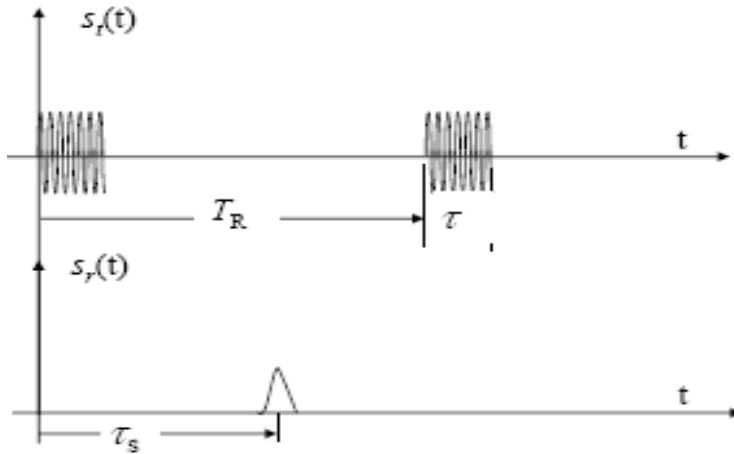
## Meteorological RADARs

- **W Band:**  
110,000-75,000 MHz; 0.27-0.4 cm
- **V,Q Band:**  
60,000-40,000 MHz; 0.5-0.8 cm
- **Ka Band:**  
40,000-26,000 MHz; 0.8-1.1 cm
- **K Band:**  
26,500-18,500 MHz; 1.1-1.7 cm
- **Ku Band:**  
12,500-18,000 MHz; 1.7-2.4 cm



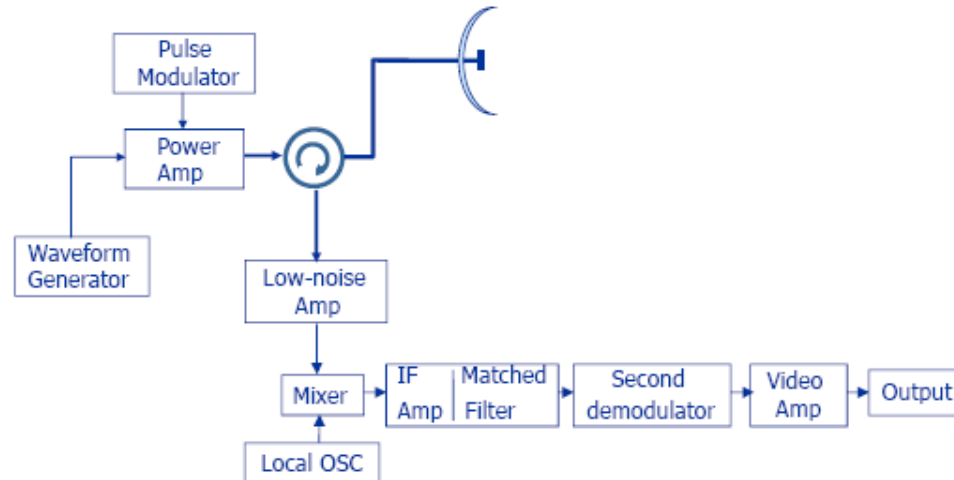
## RADAR scheme

A pulsed beam is transmitted into the atmosphere



$$r = \frac{c \cdot \tau}{2}$$

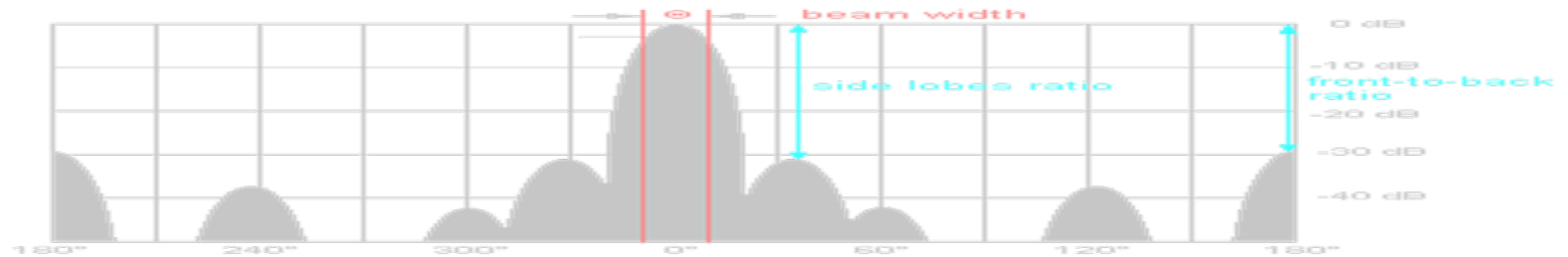
$$R_{\max} = \frac{c \cdot T_R}{2}$$



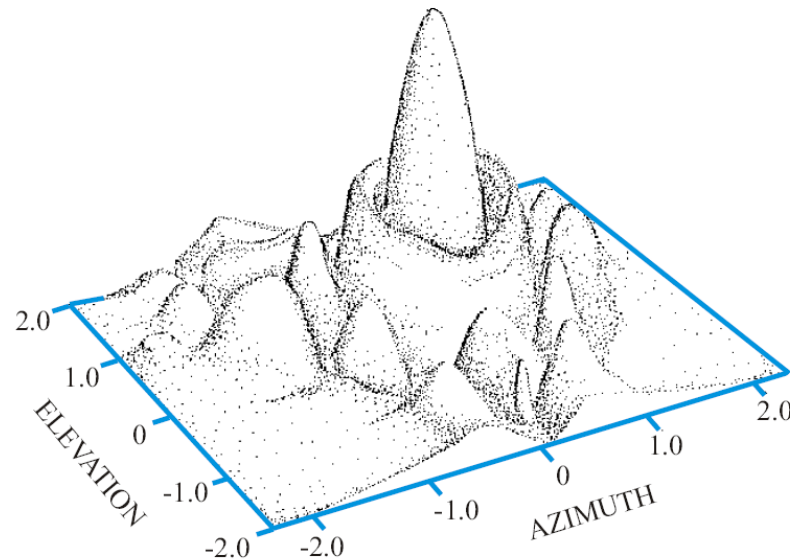
## Scattering from a far object

Electric field at distance  $r$

$$\vec{E}_i = \frac{A_i(\theta, \phi) \cdot e^{i\phi_t}}{r} \cdot e^{i(k_c \cdot r + \omega_c \cdot t)} \vec{e}_i$$



3-D Antenna Beam Pattern



**Let's suppose interaction with a single scatterer, and neglect depolarization effects, then at the radar receiving antenna:**

$$E = \frac{S(180^\circ) \cdot A(\theta_m, \varphi_m)}{k_c \cdot r_m^2} \cdot e^{i(2k_c r_m - (\omega_c - \omega_{d,m})t + \phi_{s,m} + \phi_t)}$$

**Let's assume the transmitter phase and scatterer phase shift are constant.**

## From field to power densities

**Power density over a cycle**  $P_t = \frac{1}{\eta_0} \overline{E_t^2}$

**at a distance r  
(with attenuation)**  $\frac{P_t}{4\pi \cdot r^2} f^2(\theta, \phi) \cdot G \cdot l^{-1}$

**attenuation**  $l^{-1} = e^{-\int_0^r k(r) dr}$

**At the receiver detector,  
after scattering**  $P_{r,m}(r_m, \theta_m, \phi_m) = \frac{P_t f^4(\theta_m, \phi_m) G^2 \sigma_{b,m} \lambda_c^2}{(4\pi)^2 r^4 l^2}$

$$\sigma_{b,m} = 4\pi \left( \frac{S_{2,m}(180^\circ)}{k_c} \right)^2$$

$$A_i(\varphi, \vartheta) = \sqrt{\frac{P_t G_t \eta_0}{2\pi l_t}} f_t(\varphi, \vartheta)$$

# Radar equivalent section of a reflection sphere

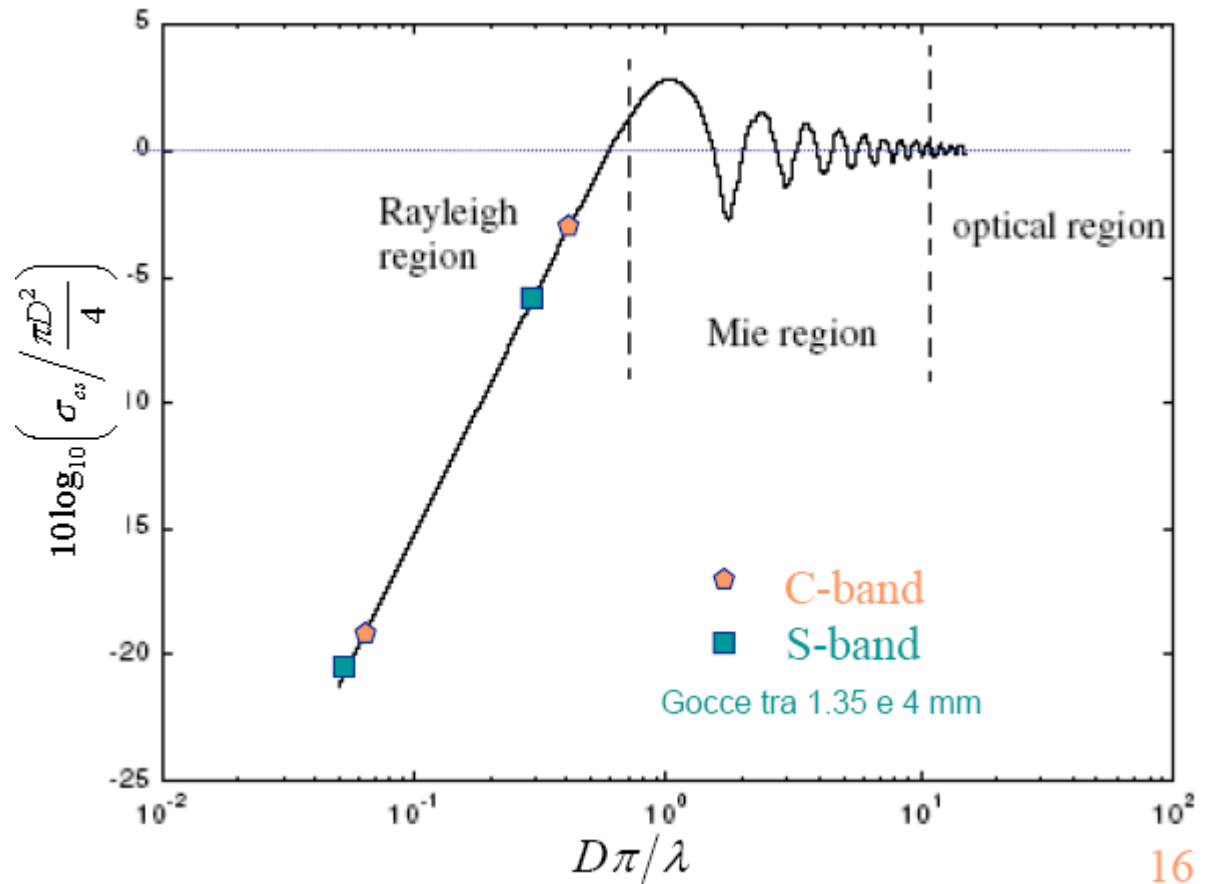
**Rayleigh region**

$$D < 10\lambda$$

$$\sigma \propto D^6/\lambda^4$$

**Optical region**

$$\sigma \propto D^2$$





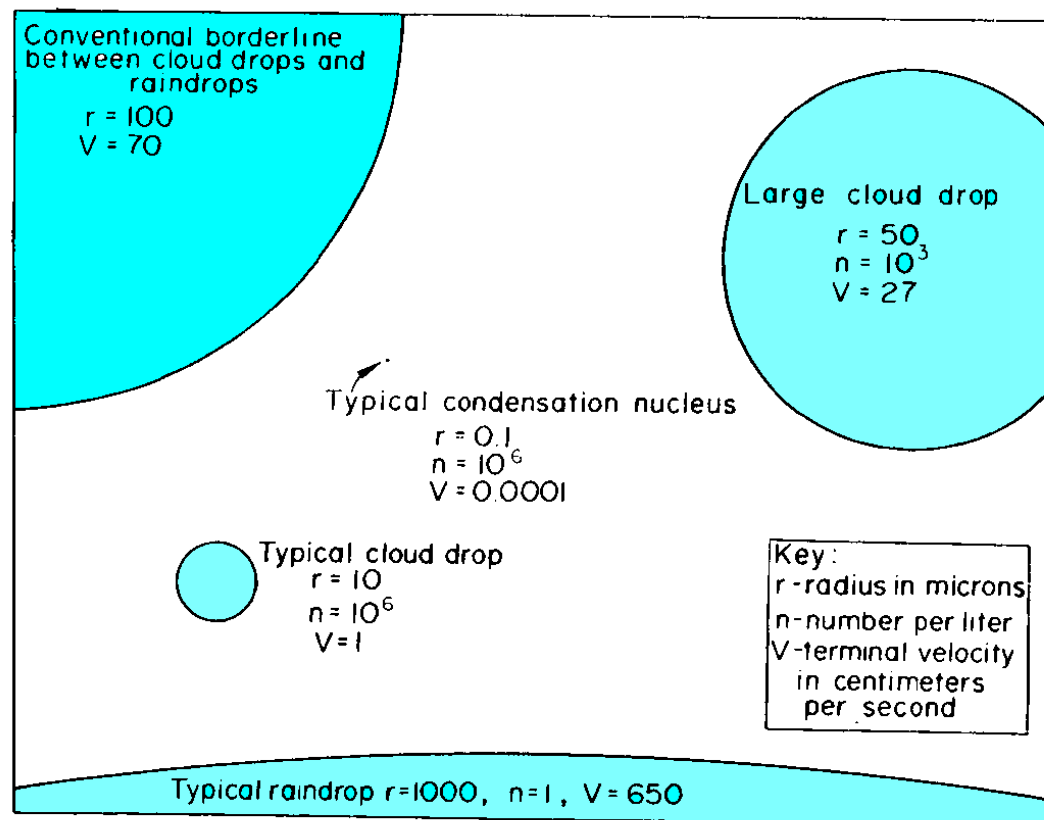


FIG. 6.1. Comparative sizes, concentrations, and terminal fall velocities of some of the particles included in cloud and precipitation processes. (From McDonald, 1958.)

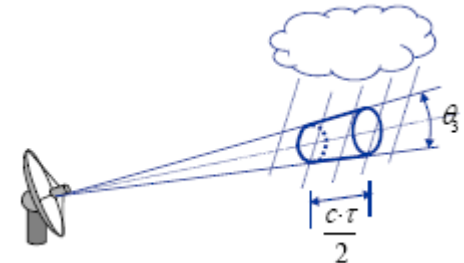
**Under the assumption of single, incoherent scatterings**

$$\sigma_{b,m} \Rightarrow \eta(r) = \int_D \sigma(D) N(D, r) dD \cdot dV$$

$$P_r(r, \theta, \varphi) = \int_{r_b}^{r_t} \int_0^\pi \int_0^{2\pi} \frac{P_t f^4(\theta, \varphi) G^2 \eta(r) dV \lambda_c^2}{(4\pi)^3 r^4 l^2}$$

**If the radar resolution length  $\tau$  and the resolution volume is small, and the antenna pattern is Gaussian and symmetric**

$$P_r(r) = \frac{P_t G^2 \lambda_c^2 \eta(r) c \tau \pi \theta^2}{(4\pi)^3 r^2 l^2(r) 16 \ln(2)} = \frac{C \eta(r)}{r^2 l^2}$$

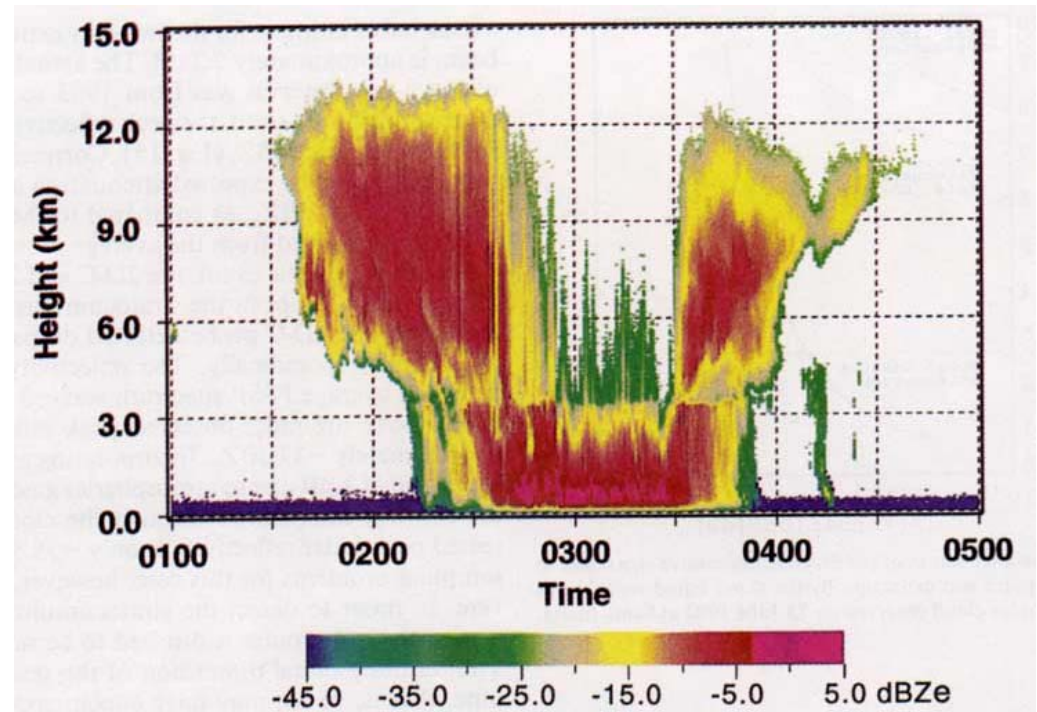


## A bit of jargon; RADAR reflectivity

$$P_r(r) = CZr^{-2}$$

**Z [mm<sup>6</sup> m<sup>-3</sup>] is the Reflectivity Factor, often expressed logarithmically**

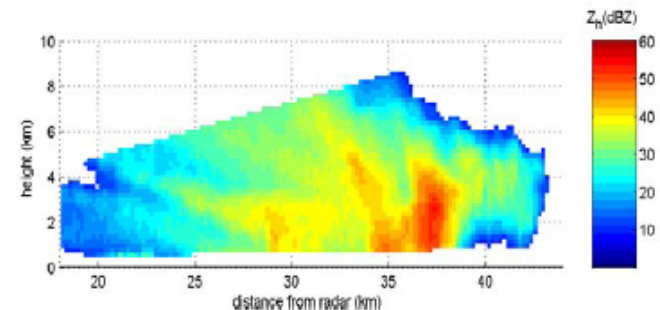
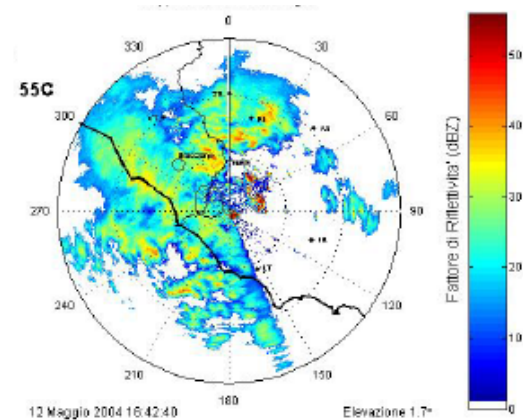
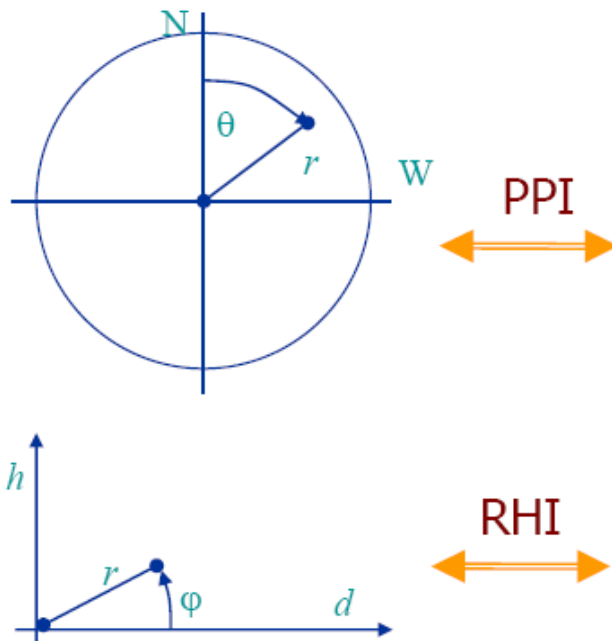
$$Z(\text{dBZ}) = 10 \log_{10}(Z)$$



Clothiaux et al., 1995

## visualization

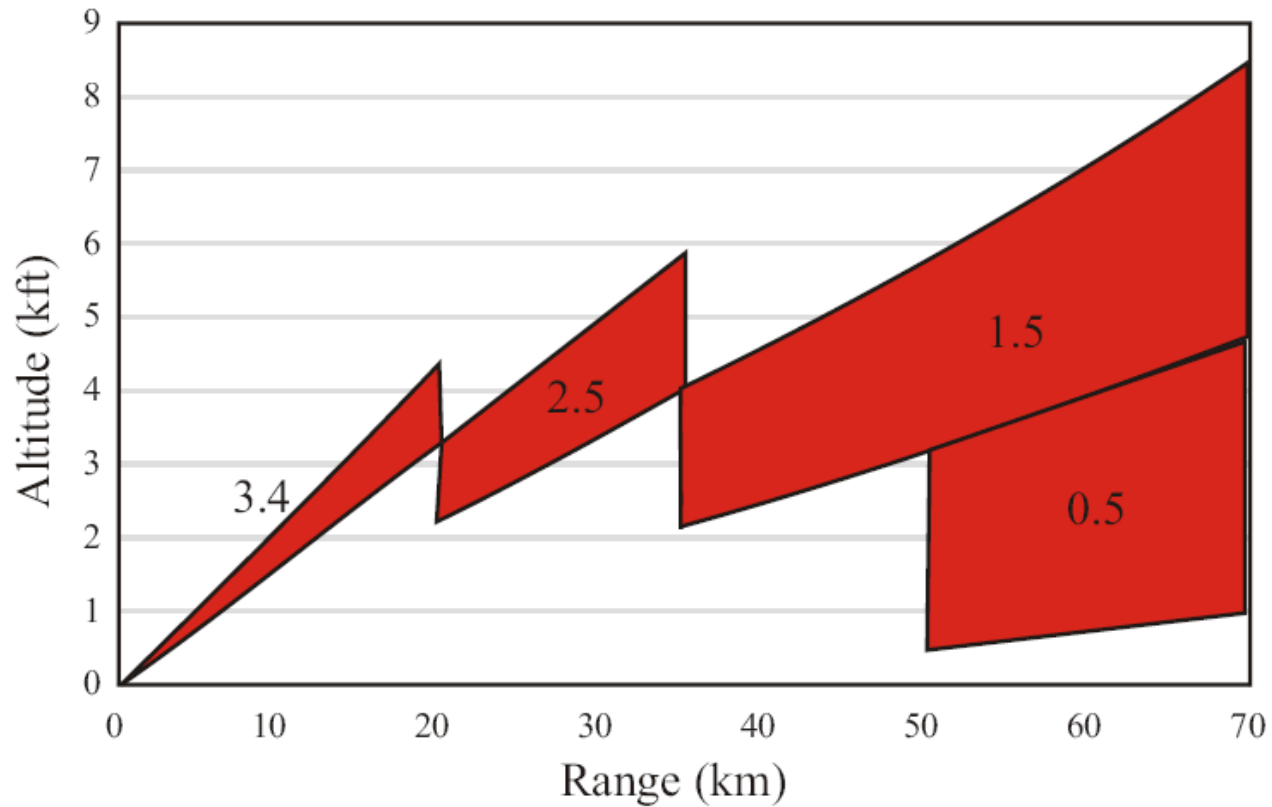
- Slant paths (Plan Position Indicator, given elevation, all azimuth),
- Vertical curtains, (Range Height Indicator, elevation scan, fixed azimuth)



Maggio 2007

Data courtesy L. Baldini

## Vertical cross sections (Constant Altitude Plan Position Indicator)



**From the antenna, the e.m. signal passes to a mixer and preamplifier, that**

- **Transform the high carrier frequency  $\omega_c$  to a lower one  $\omega_i$**
- **Produce a signal voltage proportional to the scattered field**

**From a single scatterer:** 
$$V(t) = V \cdot e^{i(2k_c r - (\omega_i + \omega_{d,m})t + \phi_{s,m} + \phi_t)}$$
$$\omega_{d,m} = 2\pi v_m / \lambda_c$$

- **Incoherent detection**
- **Coherent (Doppler) detection:**
  - **allow the determination of the phase of the received signal;**
  - **From phase variation it is possible to infer the radial velocity**

**In two I and Q demodulators, to get rid of the frequency  $\omega_i$ , the signal is multiplied by two reference voltages, a quarter out of phase, originated by a sample of the transmitted signal, then low pass filtered to obtain:**

$$I(t) = V \cos(2k_c r - \omega_{d,m} t + \phi_{s,m}) = V(\tau_i, T_s) \cos(\theta(\tau_i, T_s))$$

$$Q(t) = V \sin(2k_c r - \omega_{d,m} t + \phi_{s,m}) = V(\tau_i, T_s) \sin(\theta(\tau_i, T_s))$$

**The I and Q voltage amplitudes are a representation of the scattered e.m. field. Their Variability enable to produce power spectra. The instantaneous power at the antenna is proportional to :**

$$P(t) = C(I^2(t) + Q^2(t))$$



## from one to many scatterers (1)

**e.m. field from one scatterer:**  $E(t) = A_m \cdot e^{i[2k_c r_m - (\omega_c + \omega_{d,m})t + \phi_{s,m} + \phi_t]}$

**e.m. field from many:**  $E(t) = \sum_m A_m \cdot e^{i[2k_c r_m - (\omega_c + \omega_{d,m})t + \phi_{s,m} + \phi_t]}$

### Power received:

$$P(t) = \frac{1}{2R_0} E(t) \cdot E^*(t) = \frac{1}{2R_0} \left( \sum_{m=1,N} A_m^2 + \sum_{m \neq n, m,n=1,N} A_m A_n \cdot e^{i[2k_c (r_m - r_n) - (\omega_{d,m} - \omega_{d,n})t + (\phi_{s,m} - \phi_{s,n})]} \right)$$

$$P = \lim_{T_M \rightarrow \infty} \frac{1}{M} \sum_{j=1,M} P_j = \frac{1}{2R_0} \sum_m A_m^2$$

**The first term contains the information on the total cross section.  
The second term disappear over long averaging over many pulses.**

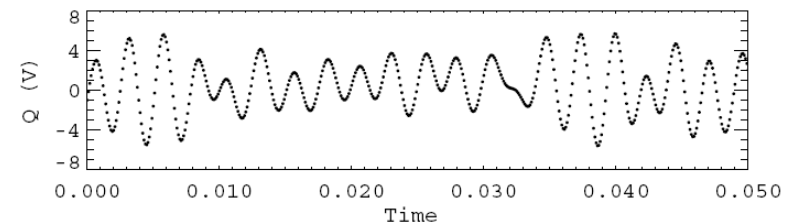
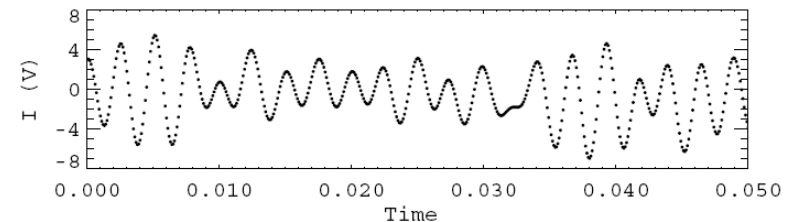
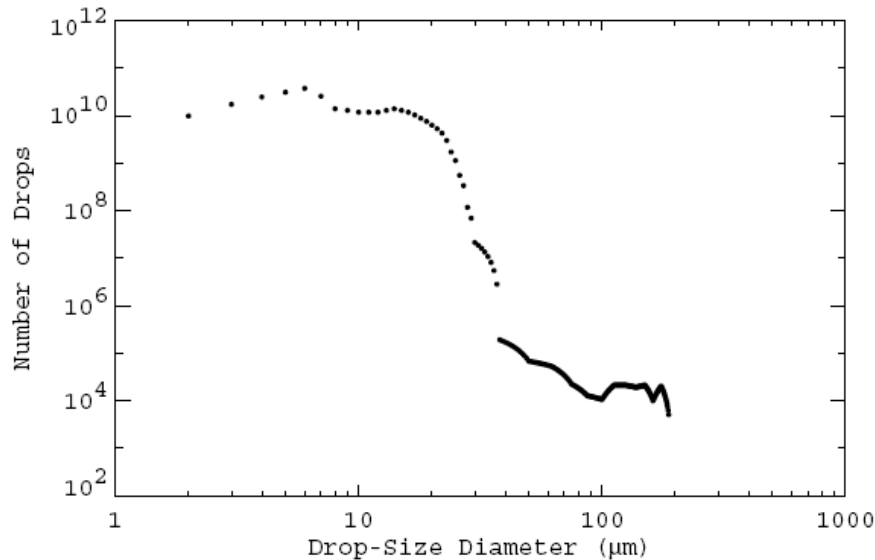


## From one to many scatterers (2)

$$I(t) = \sum_{m=1,N} V_m \cos(2k_c r_m - \omega_{d,m} t + \phi_{s,m})$$

**At the demodulators output:**

$$Q(t) = \sum_{m=1,N} V_m \sin(2k_c r_m - \omega_{d,m} t + \phi_{s,m})$$



**Simulation with a fall speed depending on the diameter**

Clotiaux et al., 2001

## Some statistical properties (1)

**The remainder of  $2k_c r_m \text{ MOD } 2\pi$  is uniformly distributed over  $0$  to  $2\pi$**

**Cosines and sines are random variables uniformly distributed between -1 and 1**

**(C. L. Th.) Their sum tends to be gaussian distributed around 0**

$$p[I(t)] = \frac{e^{-I^2(t)/2\sigma^2}}{\sqrt{2\pi}\sigma}; p[Q(t)] = \frac{e^{-Q^2(t)/2\sigma^2}}{\sqrt{2\pi}\sigma}$$

## Some statistical properties (2)

**$I(t)$  and  $I(t-L)$  show a degree of correlation if  $L$  is not too large**

**$I(t)$  and  $Q(t)$  are not correlated**

**$I(t)$  and  $Q(t+L)$  show a degree of correlation if  $L$  is not too large**

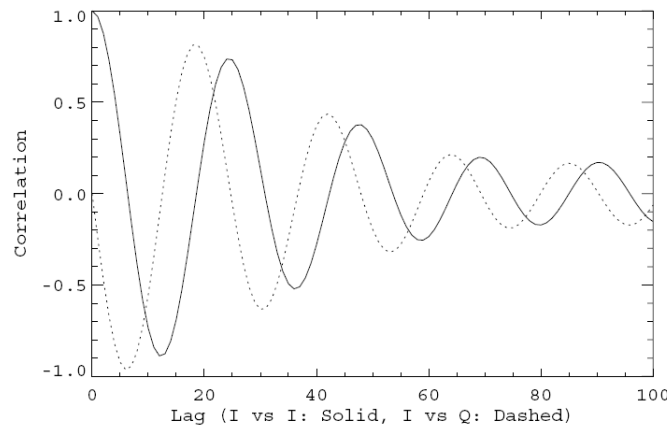


Figure 14. Correlation of  $I$  with itself (solid line) and with  $Q$  (dashed line) for lag times from 0 to  $30 T_s$ , where  $T_s = 0.0001$  is the simulated pulse repetition period for the forward simulation.

Clotiaux et al., 2001

**The instrument noise is gaussian distributed, but is uncorrelated**

## Some statistical properties (3)

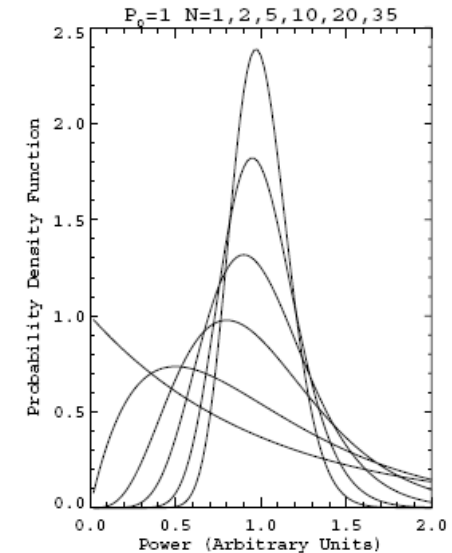
and

$$p[I(t) \cap Q(t)] = p[I(t)] \cdot p[Q(t)] = \frac{e^{-(I^2(t) + Q^2(t))/2\sigma^2}}{2\pi\sigma^2} = p[P(t)]$$

**The power probability density is exponential with peak probability 0 and mean value  $2\sigma^2$**

**And for N averages of power samples:**

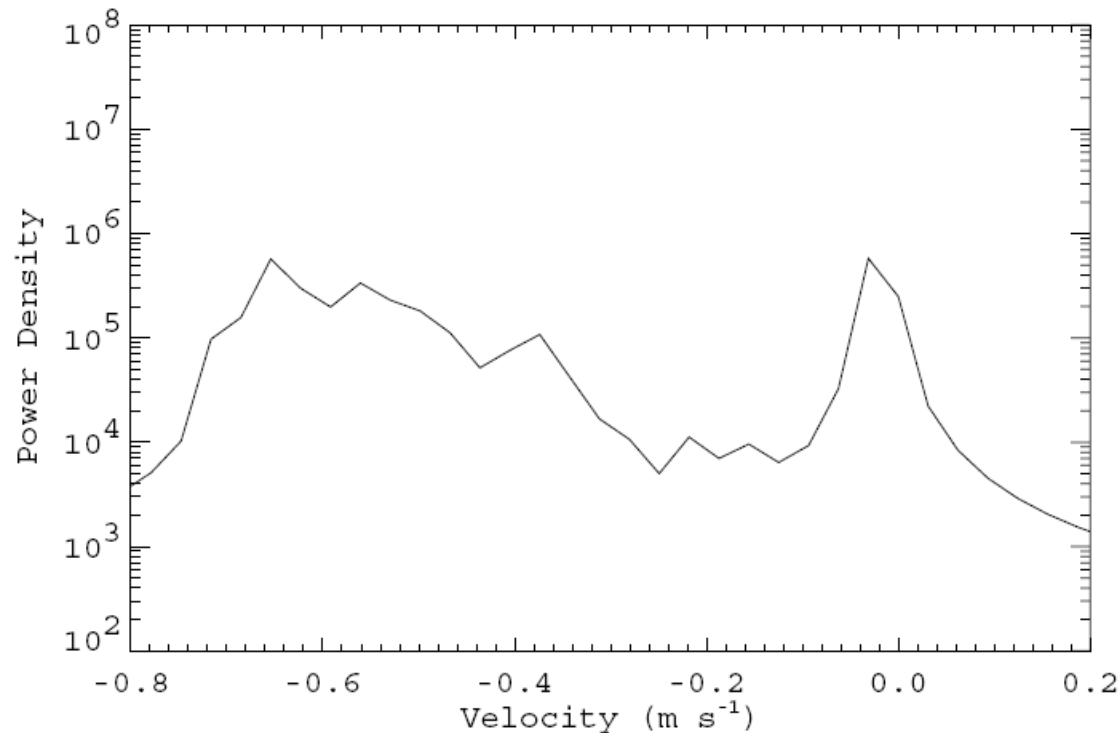
$$p[P_N] = \frac{N^N P_N^{N-1} e^{-NP_N/P_0}}{P_0^N (N-1)}$$



**From  $P_N(r)$  we retrieve informations about  $\eta(r)$ , hence the quantity and location of cloud particles.**

## Doppler moments (1)

Use  $I(t)$  and  $Q(t)$  as the real and complex array input for a FFT brings the power density spectrum  $S(\omega_m) = S(4\pi v_m / \lambda_c)$



$\tau$  fixes the max and min retrievable velocity.

T fixes the min velocity increment

## Doppler moments (2)

**Mean power weighted radiant speed**

$$\bar{v} = \sum_{-N_n/2}^{-N_n/2} v(m) \cdot S_{norm}(m)$$

**Spectral width**

$$\sigma_v^2 = \sum_{-N_n/2}^{N_n/2} \left| \bar{v} - v(m) \right|^2 \cdot S_{norm}(m)$$

**These quantities, together with the received power (zero moment) are the radar observables delivered by a Doppler system.**

### Pulse Pair autocovariance technique:

$$V(m) = I(m) + iQ(m) \quad m = 1, \dots, N$$

$$R = \frac{1}{N-1} \sum_{m=1, N} V^*(m) V(m+1)$$

$$R_{re} = \frac{1}{N-1} \sum_{m=1, N} [I(m)I(m+1) + Q(m)Q(m+1)]$$

$$R_{im} = \frac{1}{N-1} \sum_{m=1, N} [I(m)Q(m+1) - Q(m)I(m+1)]$$

$$|R| = \sqrt{R_{re}^2 + R_{im}^2}$$

$$\phi = \arctan\left(\frac{R_{im}}{R_{re}}\right)$$

## Mean backscatter-weighted radial velocity:

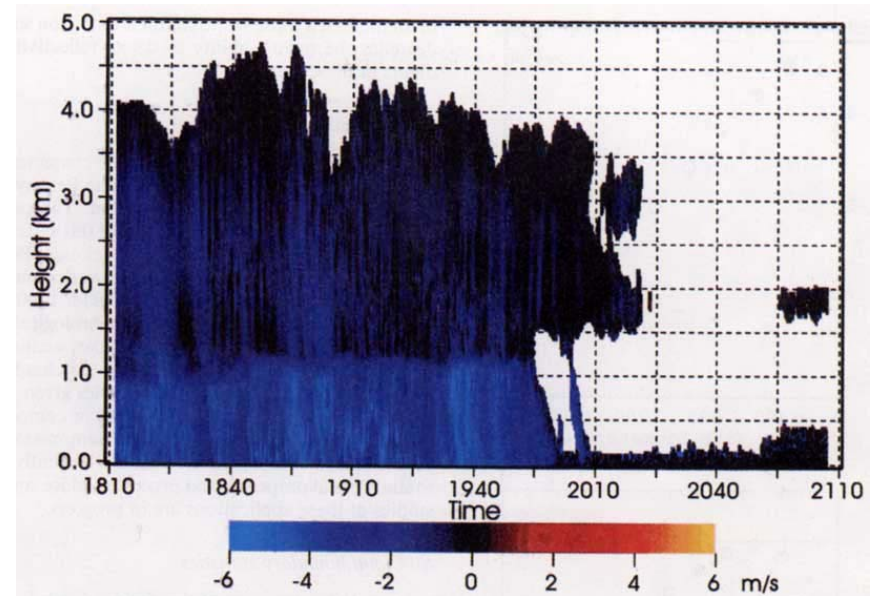
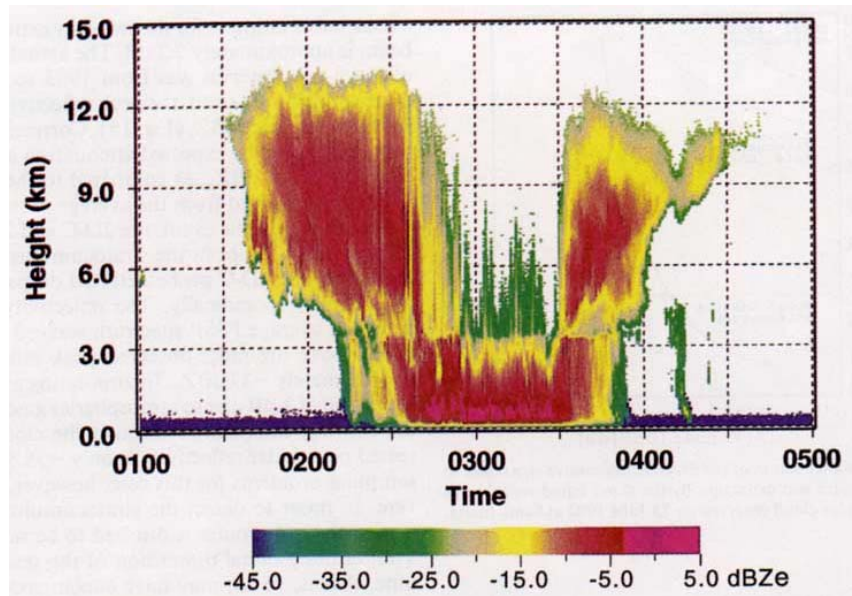
$$v = -\left(\frac{\lambda}{4\pi T}\right) \cdot \phi$$

## Spectral width:

$$\sigma_v = \left(\frac{\lambda}{2\pi T \sqrt{2}}\right) \cdot \left| \ln\left(\frac{S}{|R|}\right) \right|^{\frac{1}{2}}$$
$$S = \frac{1}{N} \sum_{m=1, N} |V(m)|^2 (-noise)$$



## Reflectivities and vertical speeds



Clothiaux et al, 1995

## ...what we have skippd

- **Z- Precipitation Intensity; - Cloud Liquid/Ice water content**
- **The effect of attenuation, negligible in S band, not so at shorter wavelenghts**
- **Georeferetiability (variability of refractive index with height)**
- **Polarimetric measurements**
- **Sources of errors; minimum detectable signal**
- **Calibration issues**
- **Etc, etc...**

## To learn more

**Bringi,V.N. and V.Chandrasekar, 2001: *Polarimetric Doppler Weather Radar: Principles and Applications*. Cambridge University Press, 636 pp.**

**Doviak D.S. and Zrnic'D. ,*Doppler radar and weather observations*. Second edition, Academic Press,1993.**

**Clothiaux et al., *Ground Based Remote Sensing of Cloud Properties using Millimeter Wave Radar*, Edited by Raschke,E., Radiation and Water in Climate System, Nato ASI Series, Springer-Verlag,1996.**